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Radix-3 Algorithm for Realization of Type-II Discrete Sine Transform

M. N. Murty* and B. Padhy**
*(Department of Physics, National Institute of Science and Technology, Palur Hills, Berhampur-761008, Odisha, India)
**(Department of Physics, Khallikote Autonomous College, Berhampur, Odisha, India)

ABSTRACT
In this paper, radix-3 algorithm for computation of type-II discrete sine transform (DST-II) of length $N = 3^m (m = 1,2, \ldots.)$ is presented. The DST-II of length N can be realized from three DST-II sequences, each of length N/3. A block diagram of for computation of the radix-3 DST-II algorithm is given. Signal flow graph for DST-II of length $N = 3^2$ is shown to clarify the proposed algorithm.
***Keywords* -** Discrete cosine transform, discrete sine transform, radix-3 algorithm.

I. INTRODUCTION

Discrete transforms play a significant role in digital signal processing. Discrete cosine transform (DCT) and discrete sine transform (DST) are used as key functions in many signal and image processing applications. There are eight types of DCT and DST. Of these, the DCT-II, DST-II, DCT-IV, and DST-IV have gained popularity. The DCT and DST transform of types I, II, III and IV, form a group of so-called “even” sinusoidal transforms. Much less known is group of so-called “odd” sinusoidal transforms: DCT and DST of types V, VI, VII and VIII.

The original definition of the DCT introduced by Ahmed *et al*. in 1974 [1] was one-dimensional (1-D) and suitable for 1-D digital signal processing. The DCT has found wide applications in speech and image processing as well as telecommunication signal processing for the purpose of data compression, feature extraction, image reconstruction, and filtering. Thus, many algorithms and VLSI architectures for the fast computation of DCT have been proposed [2]-[7]. Among those algorithms [6] and [7] are believed to be most efficient two-dimensional DCT algorithms in the sense of minimizing any measure of computational complexity.

The DST was first introduced to the signal processing by Jain [8], and several versions of this original DST were later developed by Kekre *et al.* [9], Jain [10] and Wang *et al.* [11]. Ever since the introduction of the first version of the DST, the different DST’s have found wide applications in several areas in Digital signal processing (DSP), such as image processing[8,12,13], adaptive digital filtering[14] and interpolation[15]. The performance of DST can be compared to that of the DCT and it may therefore be considered as a viable alternative to the DCT. For images with high correlation, the DCT yields better results; however, for images with a low correlation of coefficients, the DST yields lower bit rates [16]. Yip and Rao [17] have proven that for large sequence length ($N \geq 32$) and low correlation coefficient ($\rho < 0.6$), the DST performs even better than the DCT.

In this paper, a new radix-3 algorithm for computation of DST-II of length $N = 3^m (m = 1,2, \ldots.)$ is presented. The DST-II of length N is realized from three DST sequences, each of length N/3.

The rest of the paper is organized as follows. The proposed radix-3 algorithm for DST-II is presented in Section-II. An example for computation of DST-II of length $N = 3^2$ is given in Section-III. Conclusion is given in Section-IV.

II. PROPOSED RADIX-3 ALGORITHM FOR DST-II

The type-II DST for input data array $x(n), 1 \leq n \leq N,$ is defined as

$$Y(k) = \sqrt{\frac{2}{N}} C_k \sum_{n=1}^{N} x(n) \sin\left[\frac{(2n-1)k\pi}{2N}\right] \tag{1}$$

for $k = 1, 2, \ldots, N$

where ,

$$C_k = \begin{cases} \frac{1}{\sqrt{2}} \; if \; k = N \\ 1 if \; k = 1, 2, ..., N-1 \end{cases}$$

The $Y_{II}(k)$ values represent the transformed data. Without loss of generality, the scale factors in (1) are ignored in the rest of the paper. After ignoring scale factors, (1) can be written as

$$Y(k) = \sum_{n=1}^{N} x(n) \sin\left[\frac{(2n-1)k\pi}{2N}\right] \tag{2}$$

for $k = 1, 2, ..., N$.

Taking $N = 3^m \, (m = 1,2, ...)$, $Y(k)$ can be decomposed as

$$Y(3k) = \sum_{n=1}^{N/3} P(n) \sin\left[\left(\frac{2n-1}{2}\right)\frac{k\pi}{N/3}\right] \tag{3}$$

$$Y(3k-1) = \sum_{n=1}^{N/3} Q(n) \sin\left[\left(\frac{2n-1}{2}\right)\frac{k\pi}{N/3}\right] - \sum_{n=1}^{N/3} R(n) \sin\left[\left(\frac{N}{3}+k\right)\left(\frac{2n-1}{2}\right)\frac{\pi}{N/3}\right] \tag{4}$$

$$Y(N-3k+1) = \sum_{n=1}^{N/3} T(n) \sin\left[\left(\frac{N}{3}+k\right)\left(\frac{2n-1}{2}\right)\frac{\pi}{N/3}\right] + \sum_{n=1}^{N/3} W(n) \sin\left[\left(\frac{2n-1}{2}\right)\frac{k\pi}{N/3}\right] \tag{5}$$

for $k = 1,2, ..., \frac{N}{3}$.

where

$$P(n) = x(n) - x\left(\frac{2N}{3}+1-n\right)\Big) + x\left(n+\frac{2N}{3}\right) \tag{6}$$

$$Q(n) = x(n) \cos\left[\left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] - x\left(\frac{2N}{3}+1-n\right) \cos\left[\frac{2\pi}{3} - \left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] + x\left(n+\frac{2N}{3}\right) \cos\left[\frac{2\pi}{3} + \left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] \tag{7}$$

$$R(n) = x(n) \sin\left[\left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] + x\left(\frac{2N}{3}+1-n\right) \sin\left[\frac{2\pi}{3} - \left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] + x\left(n+\frac{2N}{3}\right) \sin\left[\frac{2\pi}{3} + \left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] \tag{8}$$

$$T(n) = (-1)^{n+1} Q(n) \tag{9}$$

$$W(n) = (-1)^{n+1} R(n) \tag{10}$$

for $n = 1,2, ..., \frac{N}{3}$

Define

$$C_{1n} = \cos\left[\left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] \tag{11}$$

$$C_{2n} = \cos\left[\frac{2\pi}{3} - \left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] \tag{12}$$

$$C_{3n} = \cos\left[\frac{2\pi}{3} + \left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] \tag{13}$$

$$S_{1n} = \sin\left[\left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] \tag{14}$$

$$S_{2n} = \sin\left[\frac{2\pi}{3} - \left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] \tag{15}$$

$$S_{3n} = \sin\left[\frac{2\pi}{3} + \left(\frac{2n-1}{2}\right)\frac{\pi}{N}\right] \tag{16}$$

Using (11), (12) and (13) in (7), we have

$$Q(n) = x(n) C_{1n} - x\left(\frac{2N}{3}+1-n\right) C_{2n} + x\left(n+\frac{2N}{3}\right) C_{3n} \tag{17}$$

for $n = 1,2, \dots, \frac{N}{3}$

Using (14), (15) and (16) in (8), we get

$$R(n) = x(n)S_{1n} + x\left(\frac{2N}{3} + 1 - n\right)S_{2n} + x\left(n + \frac{2N}{3}\right)S_{3n} \tag{18}$$

for $n = 1,2, \dots, \frac{N}{3}$

The type-II DSTs of $Q(n), R(n), T(n)\&W(n)$ of length $N/3$ can be expressed as

$$Y_Q(k) = \sum_{n=1}^{N/3} Q(n)\sin\left[\left(\frac{2n-1}{2}\right)\frac{k\pi}{N/3}\right] = \text{DST-II of } Q(n) \text{ of length } N/3 \tag{19}$$

$$Y_R\left(\frac{N}{3}+k\right) = \sum_{n=1}^{N/3} R(n)\sin\left[\left(\frac{N}{3}+k\right)\left(\frac{2n-1}{2}\right)\frac{\pi}{N/3}\right] = \text{DST-II of } R(n) \text{ of length } N/3 \tag{20}$$

$$Y_T\left(\frac{N}{3}+k\right) = \sum_{n=1}^{N/3} T(n)\sin\left[\left(\frac{N}{3}+k\right)\left(\frac{2n-1}{2}\right)\frac{\pi}{N/3}\right] = \text{DST-II of } T(n) \text{ of length } N/3 \tag{21}$$

$$Y_W(k) = \sum_{n=1}^{N/3} W(n)\sin\left[\left(\frac{2n-1}{2}\right)\frac{k\pi}{N/3}\right] = \text{DST-II of } W(n) \text{ of length } N/3 \tag{22}$$

for $k = 1,2, \dots, \frac{N}{3}$.

Using (19) and (20) in (4), we obtain

$$Y(3k-1) = Y_Q(k) - Y_R\left(\frac{N}{3}+k\right) \tag{23}$$

for $k = 1,2, \dots, \frac{N}{3}$.

Using (21) and (22) in (5), we have

$$Y(N-3k+1) = Y_T\left(\frac{N}{3}+k\right) + Y_W(k) \tag{24}$$

for $k = 1,2, \dots, \frac{N}{3}$.

Thus the DST-II of length $N = 3^m \, (m = 1,2, \dots)$ can be realized from three DSTs, each of length *N*/3. $Y(3k)$ in (3) is the DST of $P(n)$ of length *N/*3. It can be realized using (6). $Y(3k-1)$ in (4) can be realized using (17), (18), (19), (20) and (23). Similarly, using (9), (10), (17), (18), (21), (22) and (24), $Y(N-3k+1)$ can be realized.

The Fig.1 shows the block diagram for realization of the output components $Y(3k)$ and $Y(3k-1)$ of radix-3 DST-II of length *N*. The Fig.2 shows the block diagram for realization of the output components $Y(N-3k+1)$ of radix-3 DST-II of length *N*.

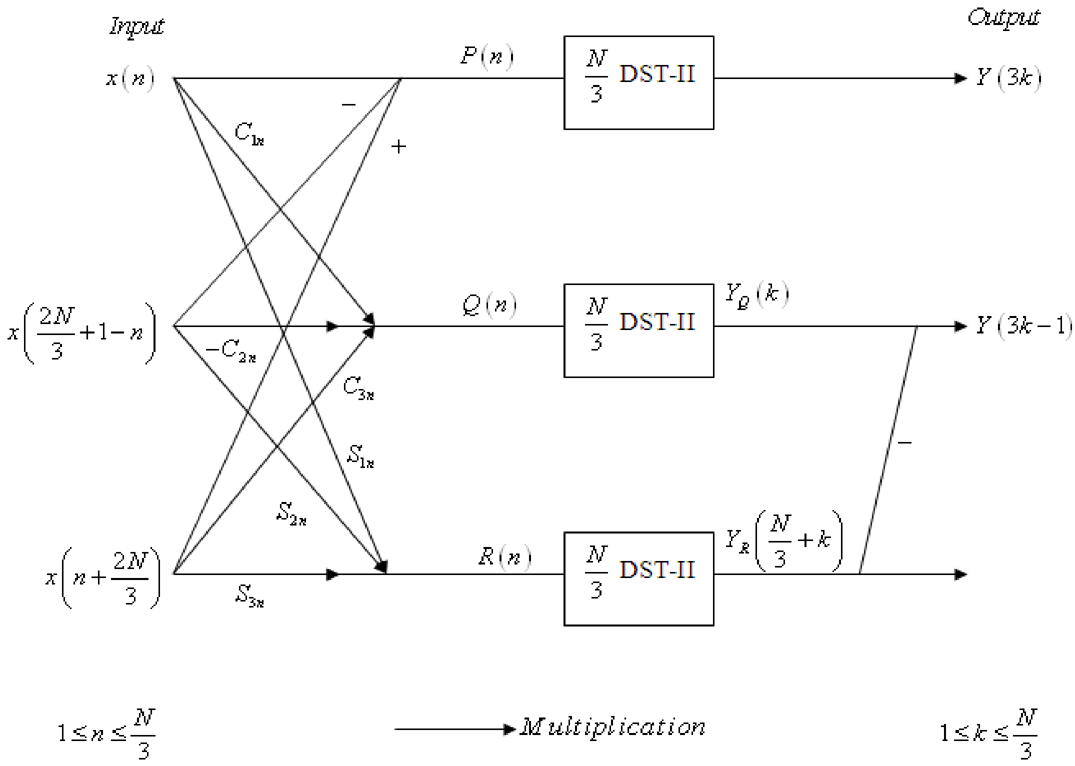

Figure 1: Block diagram for realization of output components $Y(3k)$ and $Y(3k-1)$ of radix-3 DST-II of length N .

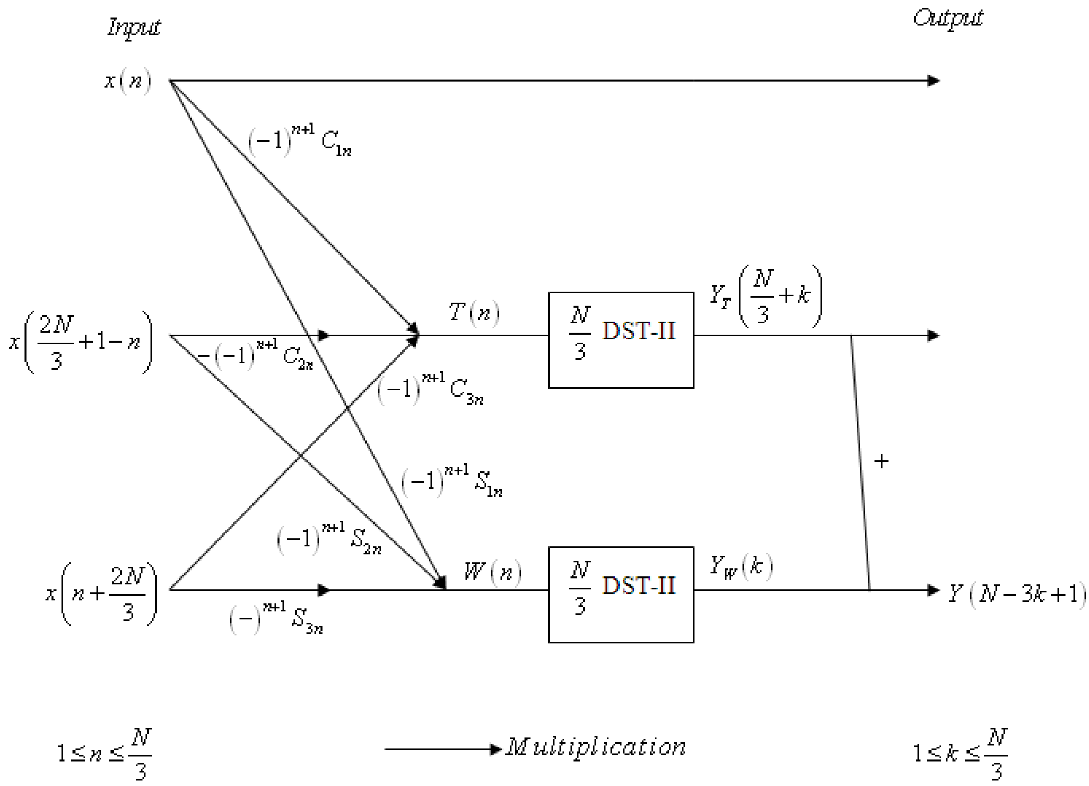

Figure 2: Block diagram for realization of output components $Y(N-3k+1)$ of radix-3 DST-II of length N .

III. EXAMPLE FOR REALIZING DST-II OF LENGTH *N* = 9

To clarify the proposed radix-3 algorithm, the DST of length $N = 3^2$ is taken. Using (6), the values of $P(n)$ for $n = 1,2$ & 3 are given by

$$P(1) = x(1) - x(6) + x(7) \tag{25}$$
$$P(2) = x(2) - x(5) + x(8) \tag{26}$$
$$P(3) = x(3) - x(4) + x(9) \tag{27}$$

The output components $Y(3), Y(6) \& Y(9)$ of DST-II are realized using (25), (26) and (27) in (3) for $k = 1,2$ & 3 and $n = 1,2$ & 3 as shown in the signal flow graph of Fig.3.

Putting successively $n = 1, 2$ & 3 in (17), the values of $Q(n)$ are given by

$$Q(1) = x(1)C_{11} - x(6)C_{21} + x(7)C_{31} \tag{28}$$
$$Q(2) = x(2)C_{12} - x(5)C_{22} + x(8)C_{32} \tag{29}$$
$$Q(3) = x(3)C_{13} - x(4)C_{23} + x(9)C_{33} \tag{30}$$

Using (18), the values of $R(n)$ for $n = 1, 2$ & 3 are given by

$$R(1) = x(1)S_{11} + x(6)S_{21} + x(7)S_{31} \tag{31}$$
$$R(2) = x(2)S_{12} + x(5)S_{22} + x(8)S_{32} \tag{32}$$
$$R(3) = x(3)S_{13} + x(4)S_{23} + x(9)S_{33} \tag{33}$$

For *k* =1, 2 and 3, we have from (23)

$$Y(2) = Y_Q(1) - Y_R(4) \tag{34}$$
$$Y(5) = Y_Q(2) - Y_R(5) \tag{35}$$
$$Y(8) = Y_Q(3) - Y_R(6) \tag{36}$$

The output components $Y(2), Y(5) \& Y(8)$ of DST-II given in (34), (35) & (36) are realized using (28) to (33) in (4) for $k = 1,2$ & 3 and $n = 1,2$ & 3 as shown in the signal flow graph of Fig.3.

Using (17) in (9), the values of $T(n)$ for $n = 1,2$ & 3 are given by

$$T(1) = x(1)C_{11} - x(6)C_{21} + x(7)C_{31} \tag{37}$$
$$T(2) = -x(2)C_{12} + x(5)C_{22} - x(8)C_{32} \tag{38}$$
$$T(3) = x(3)C_{13} - x(4)C_{23} + x(9)C_{33} \tag{39}$$

Using (18) in (10), the values of $W(n)$ for $n = 1, 2$ & 3 are given by

$$W(1) = x(1)S_{11} + x(6)S_{21} + x(7)S_{31} \tag{40}$$
$$W(2) = -x(2)S_{12} - x(5)S_{22} - x(8)S_{32} \tag{41}$$
$$W(3) = x(3)S_{13} + x(4)S_{23} + x(9)S_{33} \tag{42}$$

For *k* = 1, 2 and 3, we obtain from (24)

$$Y(7) = Y_T(4) + Y_W(1) \tag{43}$$
$$Y(4) = Y_T(5) + Y_W(2) \tag{44}$$
$$Y(1) = Y_T(6) + Y_W(3) \tag{45}$$

The output components $Y(7), Y(4) \& Y(1)$ of DST-II given in (43), (44) & (45) are realized using (37) to (42) in (5) for $k = 1,2$ & 3 and $n = 1,2$ & 3 as shown in the signal flow graph of Fig.4.

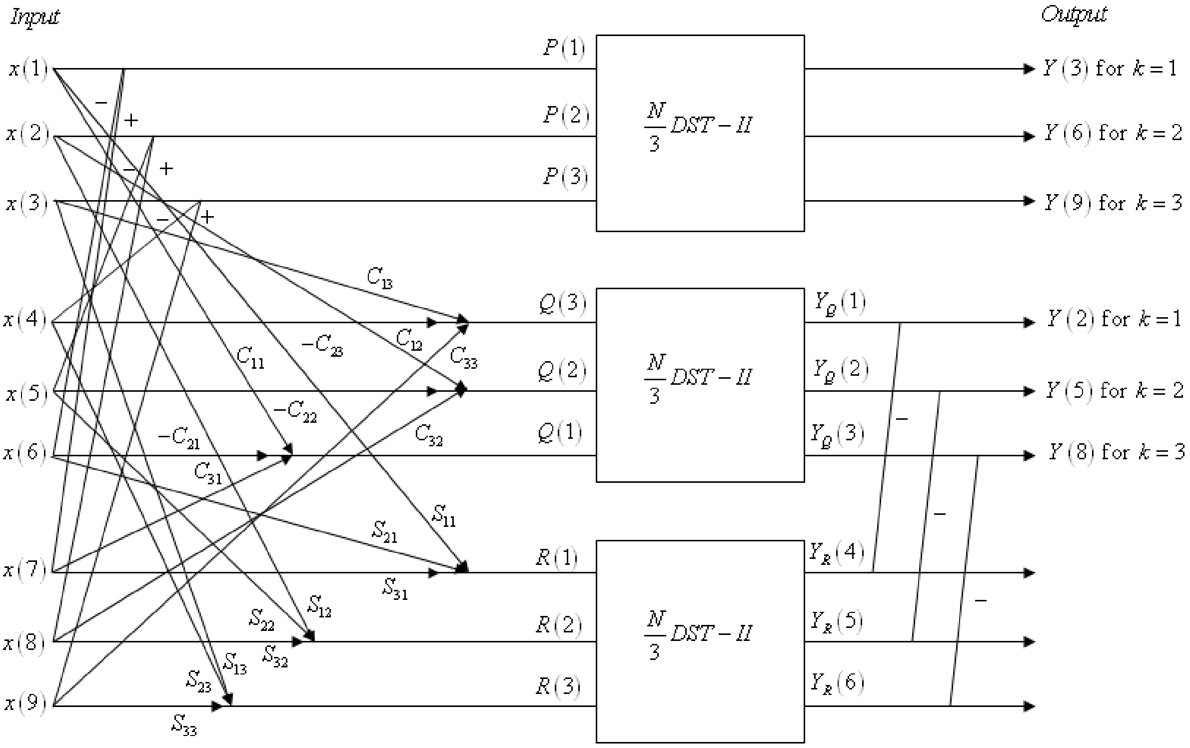

Figure 3: Signal flow graph for realization of the output components $Y(2), Y(3), Y(5), Y(6), Y(8)$ and $Y(9)$ of radix-3 DST-II of length *N*=9.

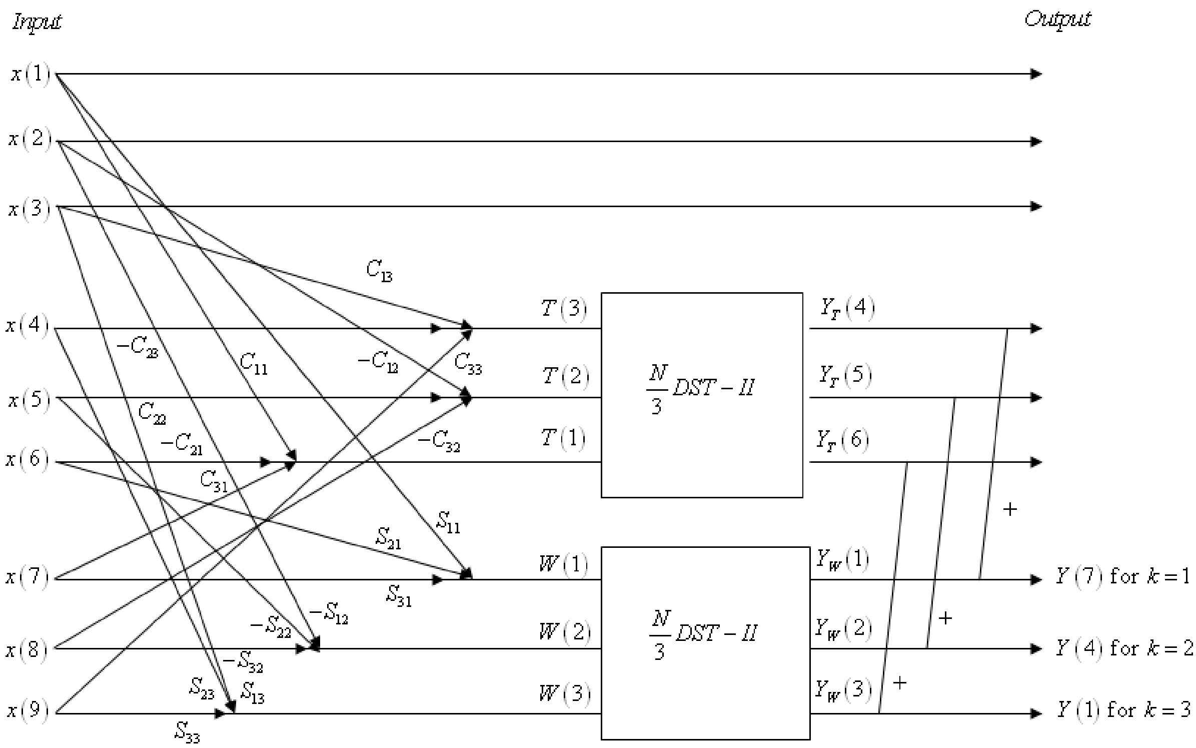

Figure 4: Signal flow graph for realization of the output components $Y(1), Y(4),$ and $Y(7)$ of radix-3 DST-II of length *N*=9.

IV. CONCLUSION

A new radix-3 algorithm for realization of DST-II of length $N = 3^m (m = 1,2, \dots.)$ has been presented. A block diagram for implementation of this algorithm is shown and the signal flow graph for DST of length $N = 3^2$is given to clarify the proposal. This algorithm can easily be implemented due its simple and regular structure.

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